SUSTAIN

GAME-BASED LEARNING ON URBAN SUSTAINABILITY

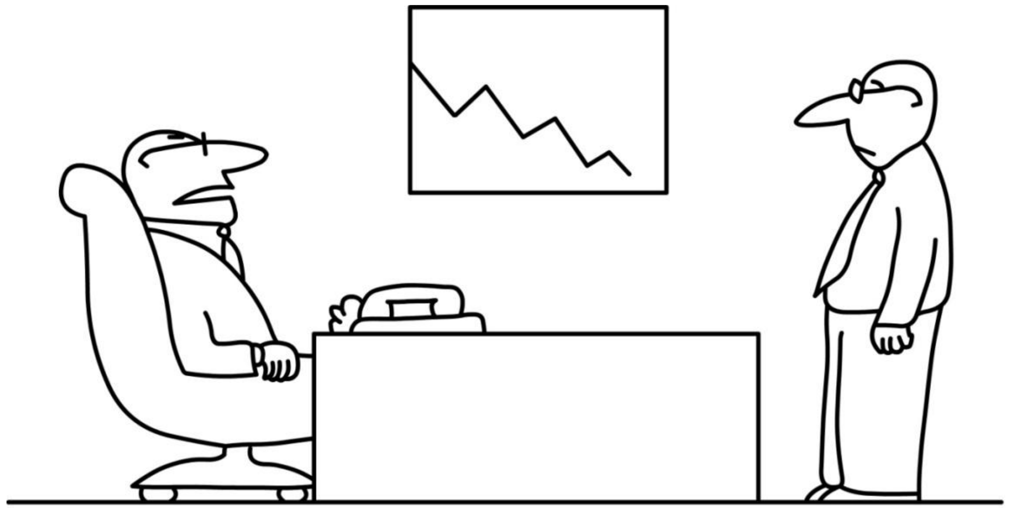
SUSTAIN Project - Funded under the Erasmus+ KA2- 2017-1-EL01-KA203-036303

Co-funded by the Erasmus+ Programme of the European Union

ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ
IKY

A short introduction to Operational Research

Linear Programming and Data Envelopment Analysis



©2008 www.timoelliott.com

"It would appear, Hopkins, that your gut feel was only indigestion"

SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY
Co-funded by the Erasmus+ Programme of the European Union
ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY
System Dynamics Italian Chapter
ERGO LVDO EDITIONS
CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA
60 YEARS 1957-2017
Societatea pentru Consum Responsabil
Centre for Systems Solutions

Problem Solving

Problem Solving is a process:

- *Identify a gap between the state of a system as it is and as it is desired to be*
- *Take the necessary measures to make the gap smaller*

State of system

Desired State of system

SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY

Co-funded by the Erasmus+ Programme of the European Union

ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY

Problem Solving

1) *Identify and define the problem*
2) *Determine the set of alternative actions*
3) *Determine the criteria that will be used for the evaluation of the alternatives*

State of system

↓

Desired State of system

SYSTEM DYNAMICS ITALIAN CHAPTER

ERGO LVDO EDITIONS

CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS

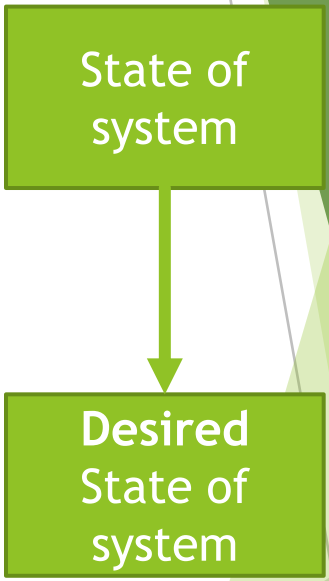
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA 60 YEARS 1957-2017

Societatea pentru Consum Responsabil

Centre for Systems Solutions

Problem Solving

4) Perform the evaluation of the alternatives
5) Make the choice
6) Implement the chosen alternative
7)Evaluate the impact of the chosen alternative to the perceived gap

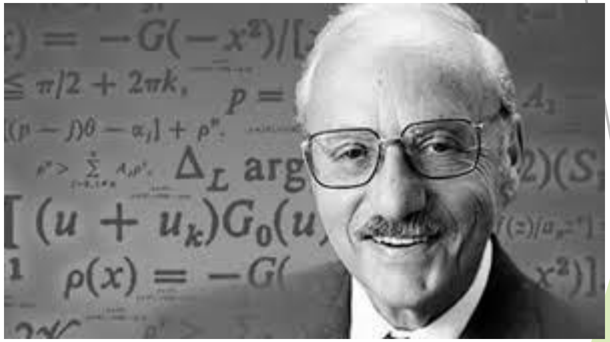


SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY
Co-funded by the Erasmus+ Programme of the European Union
ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY
SYSTEM DYNAMICS ITALIAN CHAPTER
ERGO LVDO EDITIONS
CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA 60 YEARS 1957-2017
Societatea pentru Consum Responsabil
Centre for Systems Solutions

Linear Programming

LP deals with problems where there is a need to optimize resources in order to perform activities that are competing with each other

Linear Programming



SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY
Co-funded by the Erasmus+ Programme of the European Union

Linear Programming

The word “program” in Linear Programming is a historical artifact. The initial problem was a resource allocation problem for the Air Force of the United States of America. The decisions about resource allocation were called “Programs” and the term remained ever since

CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA
Societatea pentru Consum Responsabil
Centre for Systems Solutions

Linear Programming

Objective of LP:

Maximize or minimize a quantity taking into account restriction that limit the degree to which the objective can be achieved

Linear Programming

Criteria of LP problem:

- The variables that represent the competing activities (or services, products etc.) that the decision maker can act upon. They are **called decision variables**.
- The function that represents the quantity that needs maximization or minimization is called **objective function**.
- The restrictions that limit the degree to which the objective can be achieved are called **constraints**.
- Finally, there is the solution of the LP problem, if it does not violate any of the constraints is called feasible solution. The set of all feasible solutions is called the **feasible area** of the problem. If a feasible solution also maximizes or minimizes the objective function, it is called **optimal**

Linear Programming

Maximization Problem

The owner of a small telecommunications company manufactures two types of mobile phones, phone A and phone B. The company has set the price at such a level that for each phone A that is being sold the profit is 150€, while for phone B is 200€. However, the company cannot manufacture an infinite amount of phones. The government with a new law that aims to protect the working force has declared that only 550 working hours will be available to the company per month (based on the number of its employees). To manufacture both phones, 1 working hour for each is required. Furthermore, for the construction of the phones a very rare mineral is required and the company has only a limited amount of it, 2000 mg. To manufacture phone A, 5 mg of the mineral are required while for phone B 2 mg.

Co-funded by the Erasmus+ Programme of the European Union
ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY

Linear Programming

Maximization Problem

Moreover, the company produces greenhouse emissions when constructing the phones. An analyst has calculated that 1 μg of harmful emissions are emitted for every phone A that is constructed and 3μg for every phone B. According to EU regulations though, the company cannot emit more than 1000 μg of harmful gases per month in order to protect the environment. Finally, the CEO of the company has decided that this will be the last time that the company produces phone A and they should not produce more than 280 phones A. The problem is to help management determine how many of each phone they should produce in order to maximize their profits, while respecting all the constraints. The above problem is a typical LP problem. The following paragraphs will present a series of steps on how to solve such problems. The first step is to translate every important piece of information into a mathematical term or function.

Linear Programming

Maximization Problem

Decision variables:

The variables that the decision maker can act upon are the numbers of each phone that will be produced

Linear Programming

Maximization Problem

Decision variables:

$$x_1 = number\ of\ phones\ A$$
$$x_2 = number\ of\ phones\ B$$

SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY
Co-funded by the Erasmus+ Programme of the European Union
ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY

Linear Programming

Maximization Problem

Objective Function (Maximize Profit)

$$P = 150 * x_1 + 200 * x_2$$

SYstem Dynamics Italian Chapter
ERGO LVDO EDITIONS
CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA
60 YEARS 1957-2017
Societatea pentru Consum Responsabil
Centre for Systems Solutions

Linear Programming

Maximization Problem

Constraints

1) The working hours for constructing the phones must be less than or equal to the number of hours defined by the government. In mathematical terms:

$$1 * x_1 + 1 * x_2 \leq 550$$

2) The use of the mineral must be less that the quality that the company possesses. In mathematical terms:

$$5 * x_1 + 2 * x_2 \leq 2000$$

Linear Programming

Maximization Problem

Constraints

3) Emissions produced by the company must be lower that the limit imposed by the EU. In mathematical terms:

$$1 * x_1 + 3 * x_2 \leq 1000$$

4) The number of phones A must not be more that the limit set by the president of the company. In mathematical terms:

$$x_1 \leq 280$$

Linear Programming

$$Maximize$$
$$P = 150 * x_1 + 200 * x_2$$

$$subject\ to\ the\ constraints:$$

$$1 * x_1 + 1 * x_2 \leq 550$$
$$5 * x_1 + 2 * x_2 \leq 2000$$
$$1 * x_1 + 3 * x_2 \leq 1000$$
$$x_1 \leq 280$$
$$x_1, x_2 \geq 0$$

Linear Programming

ASSUMPTIONS FOR LINEAR PROGRAMMING

There are 4 assumptions that are necessary in order for a LP model to be appropriate:

Proportionality: It means that the contribution to the objective function from a decision variable must be linear. The same applies for the constraints. Thus, an objective function of the form $x_2 + x_1^2 + 3$ cannot be part of an LP problem.

Additivity: there must not be a relationship among the decision variables. Thus, it is not an LP problem if it has the form $x_2 * x_1 + x_1^2$

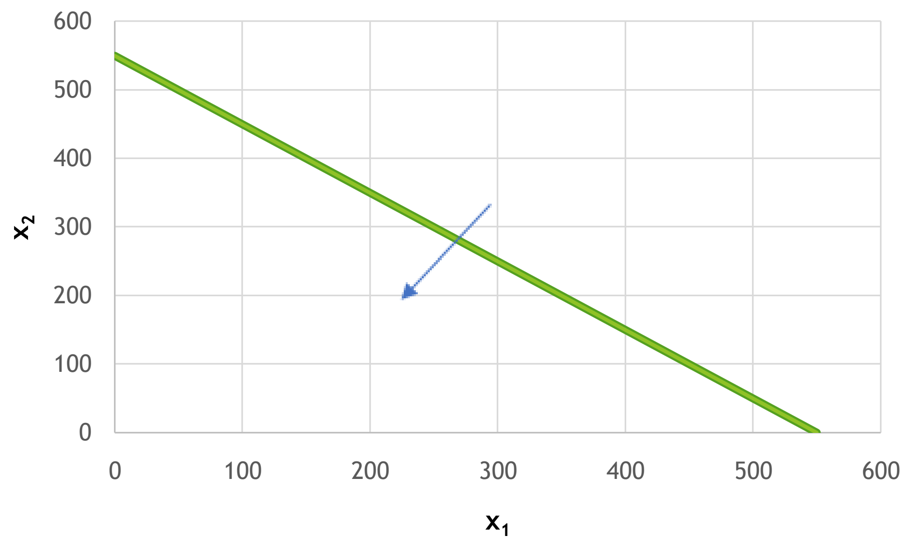
Divisibility: All variables must be continuous

Certainty: All data must be known and precise

Linear Programming

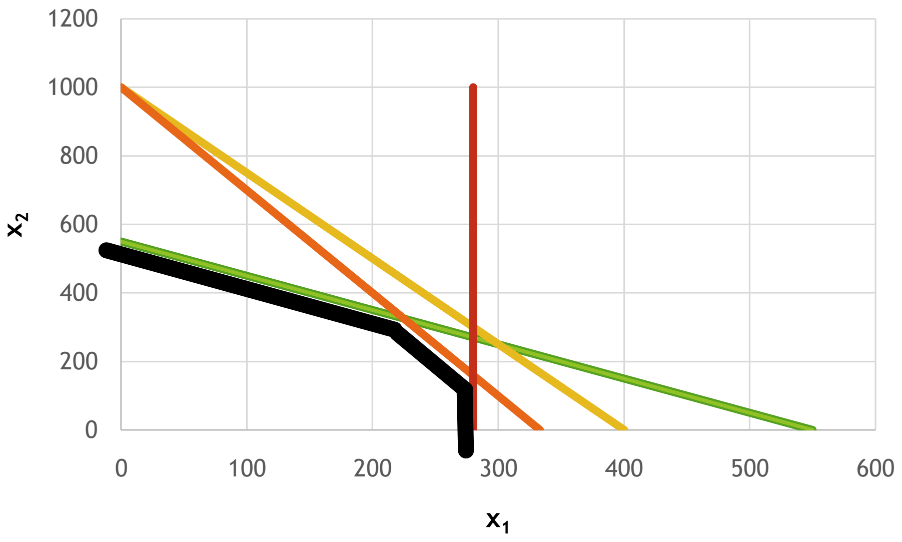
From Constraint 1

$$x_1 = 550 - x_2$$



Linear Programming

From Constraints



Linear Programming

Extreme points of the feasible region are the solutions to the LP

X_1	X_2	P
0	0	0
0	550	110000
225	325	98750
280	160	74000
280	0	42000

Linear Programming

Questions:

- Knowing the optimal values for x_2 and x_1, what happens to the other constraints? Are all the resources used (time, the limit for emissions)?
- Can you provide an interpretation of your answer to question a?

Co-funded by the Erasmus+ Programme of the European Union

Linear Programming

Exercise

In board games there might be a time where it is necessary to place some resources (workers, pawns, buildings etc.) in places on the board. However, each worker placement comes with a cost and you only have a specific number of workers (resources) at your disposal. For example, let's assume that you are the mayor of a city and you need to decide on the construction of 3 new bus terminals (named B1, B2, B3). There are four candidate sites S1, S2, S3, S4. However there is also an expected cost for the placement of a terminal to any of the spots. The table below presents these costs.

	S1	S2	S3	S4
B1	13	16	12	11
B2	15	2	13	20
B3	5	7	10	6

However, in this problem each terminal can only be assigned to one site and each site can only receive/accommodate only one terminal.

- What are the decision variables?
- What is the mathematical formulation of the problem?

Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a non-parametric, mathematical programming technique that is used for the evaluation of the technical efficiency of **Decision Making Units (DMUs)** relative to one another.

By non-parametric it is meant that there are no assumptions as to the forms of the parameters, the form of the frequency distributions etc.
DMUs are entities such as people, firms, organizations (public and private) that convert multiple inputs to multiple outputs.

In DEA the DMUs are considered homogeneous in terms of identical inputs and outputs

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

Technical Efficiency

$$technical\ efficiency = \frac{\sum w_{output} * y}{\sum w_{input} * x}, where\ x = input\ level\ and\ y = output\ level$$

Data Envelopment Analysis

Technical Efficiency

The definition of technical efficiency allows the comparative assessment of DMUs in situations where price information is not available

Data Envelopment Analysis

Technical Efficiency

As a result, DEA is considered a suitable method for:

- aggregating economic, social and environmental factors with different units with the purpose of constructing a multi-dimensional indicator of efficiency.
- This indicator can help a decision-maker to comprehend better which DMUs under his control are efficient, where there is room for improvement and how this improvement can be achieved

SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY
Co-funded by the Erasmus+ Programme of the European Union
ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY
SYstem Dynamics Italian Chapter
ERGO LVDO EDITIONS
CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA
60 YEARS 1957-2017
Societatea pentru Consum Responsabil
Centre for Systems Solutions

Data Envelopment Analysis

Problem Formulation

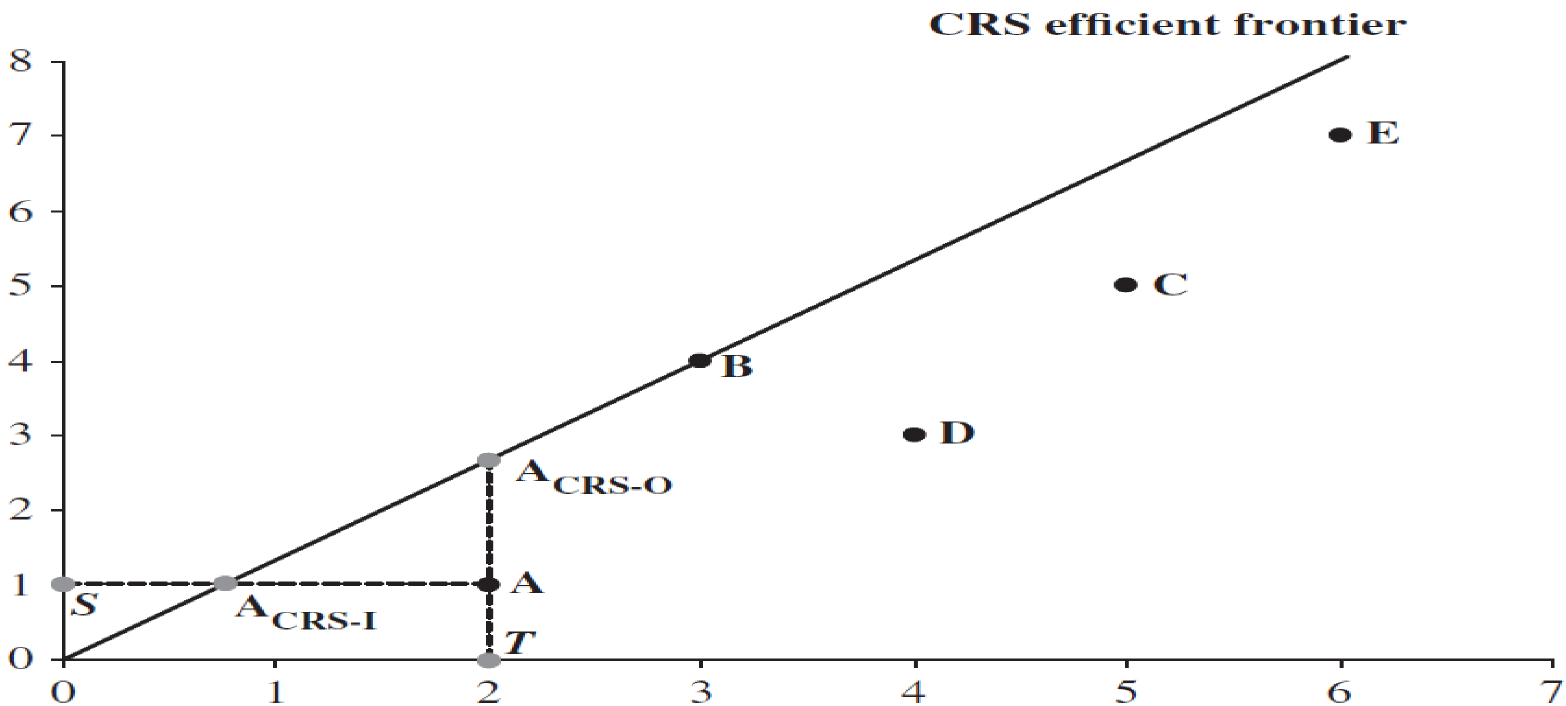
- Suppose there are N homogeneous DMUs, where each DMU uses m inputs to produce s outputs.
- Identify the most efficient DMU by maximizing each one's individual efficiency.
- Each DMU's efficiency is calculated relative to an efficient frontier
- DMUs located at the efficiency frontier has an efficiency score of 1 (or 100%), while those under the frontier have an efficiency of less than 1 (or 100%), hence they have the capacity to improve

SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY
Co-funded by the Erasmus+ Programme of the European Union
ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY
SYstem Dynamics Italian Chapter
ERGO LVDO EDITIONS
CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA
60 YEARS 1957-2017
Societatea pentru Consum Responsabil
Centre for Systems Solutions

Data Envelopment Analysis

Problem Formulation

- Constant Returns to Scale



Data Envelopment Analysis

Problem Formulation

- Variable Returns to Scale

Increasing returns to scale: In this situation, the average inputs consumption decreases while output rises. For example, a change in the output of 10% results in change in inputs of less than 10%. For a DMU, operating in increasing returns to scale and wish to improve, it means an expansion of output is necessary.

Decreasing returns to scale: In this situation, the average inputs consumption increases while output rises. For example, a change in output of 10% results in change in inputs of more than 10%. When a DMU is operating under decreasing returns to scale and wish to improve, it means a reduction of output.

Data Envelopment Analysis

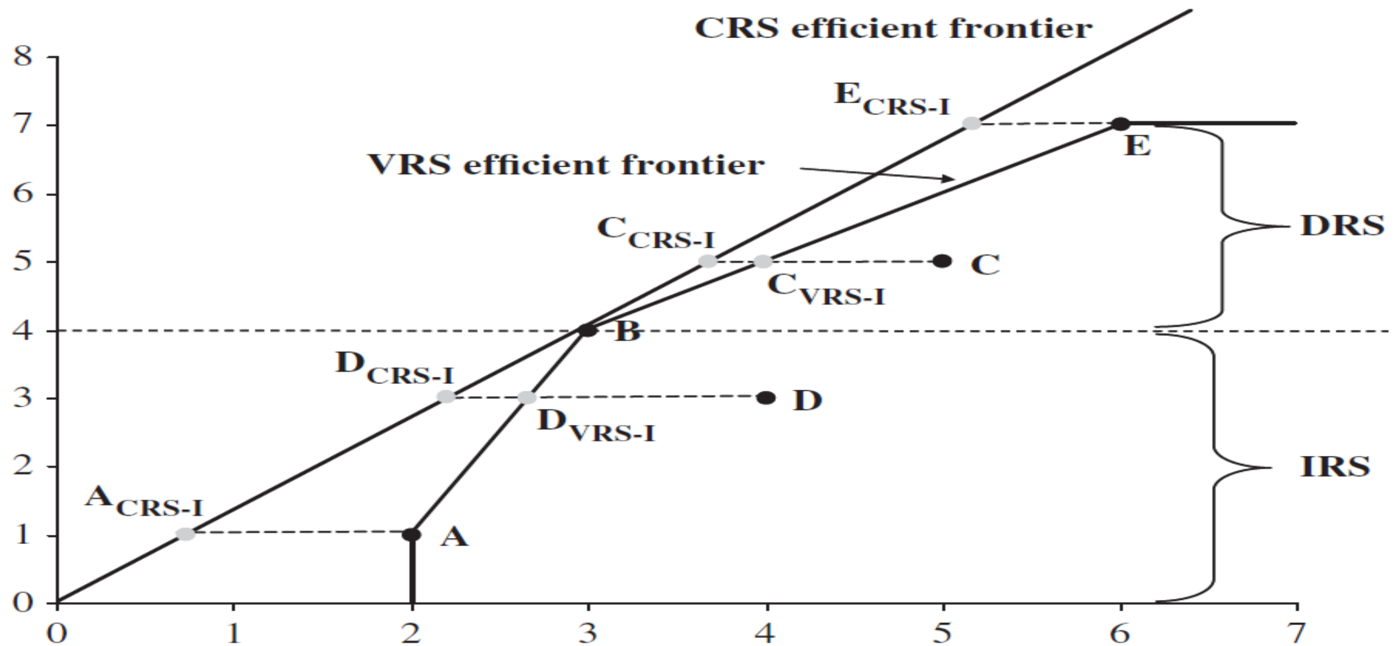
EXAMPLE

Let us assume that in a potential board game there are 5 mines in which each player must place some workers. The mines are represented by the nodes labeled A to E. The x-axis represents the number of workers (input) and the y-axis the kilograms (kg) of precious metal that they excavated in each mine (output). For example, in mine A, 2 workers have been placed and the excavated 1 kg of precious metals, in mine B 3 workers have been placed that excavated 4 kg of precious metals etc.

To evaluate the efficiency of each mine all that is necessary is to check its position with regards to the frontiers and contemplate on the nature of scales. To identify the nature of scale, focus on the slope of the VRS efficient frontiers (or the projection of VRS inefficient points). In figure below, the projections of mines D and C in the VRS frontier are the points $D_{vrs\text{-}i}$and $C_{vrs\text{-}i.}$

Data Envelopment Analysis

EXAMPLE



Data Envelopment Analysis

EXAMPLE

In an Input orientation we are trying to minimize the input to produce the same level of output

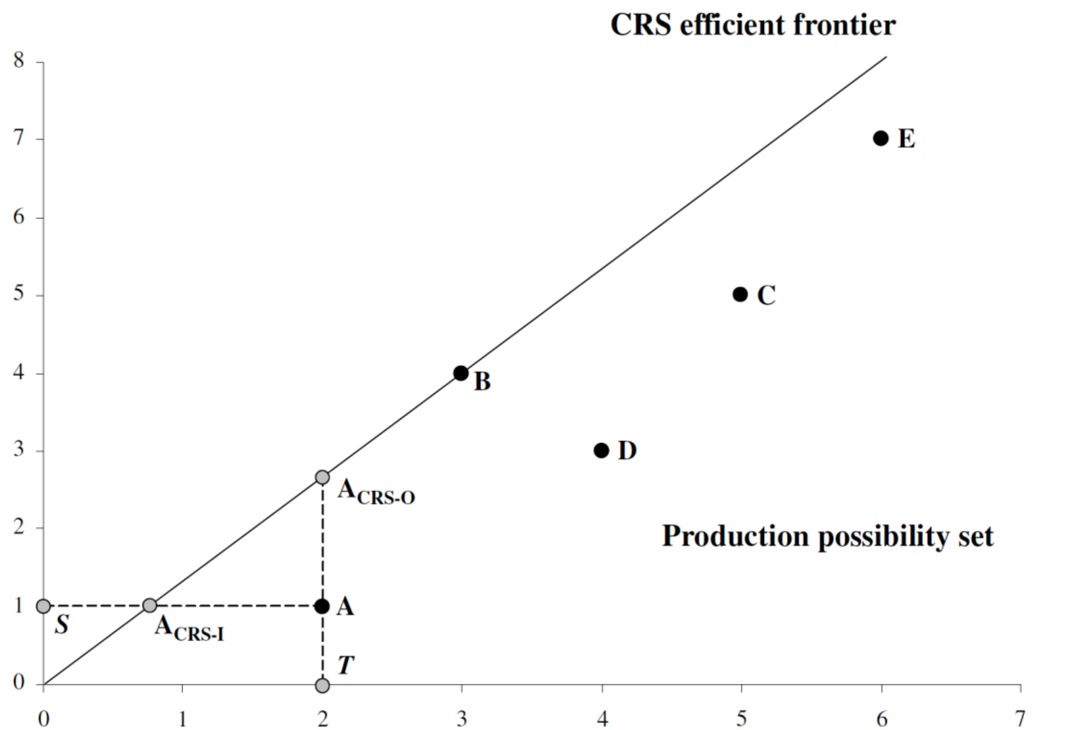
Data Envelopment Analysis

EXAMPLE

Technical Efficiency is measured by:

Distance(SA_{CRSI})/distance (SA) = 0.75/2 = 0.375

Technical Efficiency of A = 37.5%

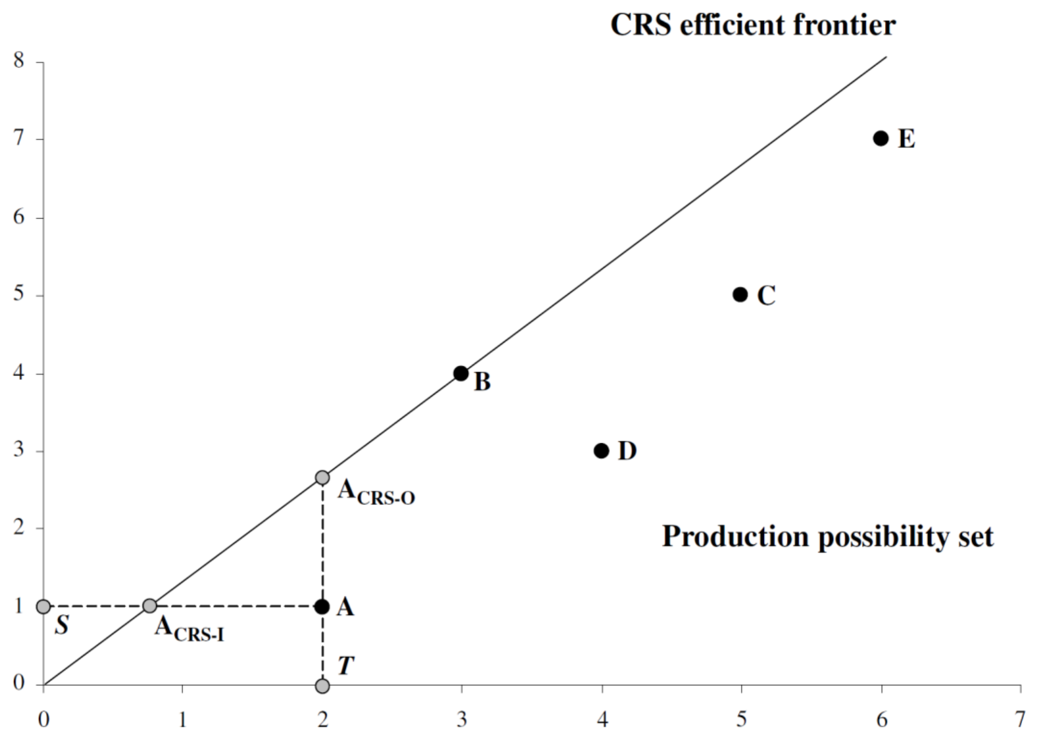


Data Envelopment Analysis

EXAMPLE

Technical Efficiency of A = 37.5%

It means that mine A could reduce the number of workers on the mine by 62.5% (100-37.5%) and still be able to produce the same number of mineral



Data Envelopment Analysis

EXAMPLE

In an Output orientation we are trying to maximize the output by keeping the same level of input

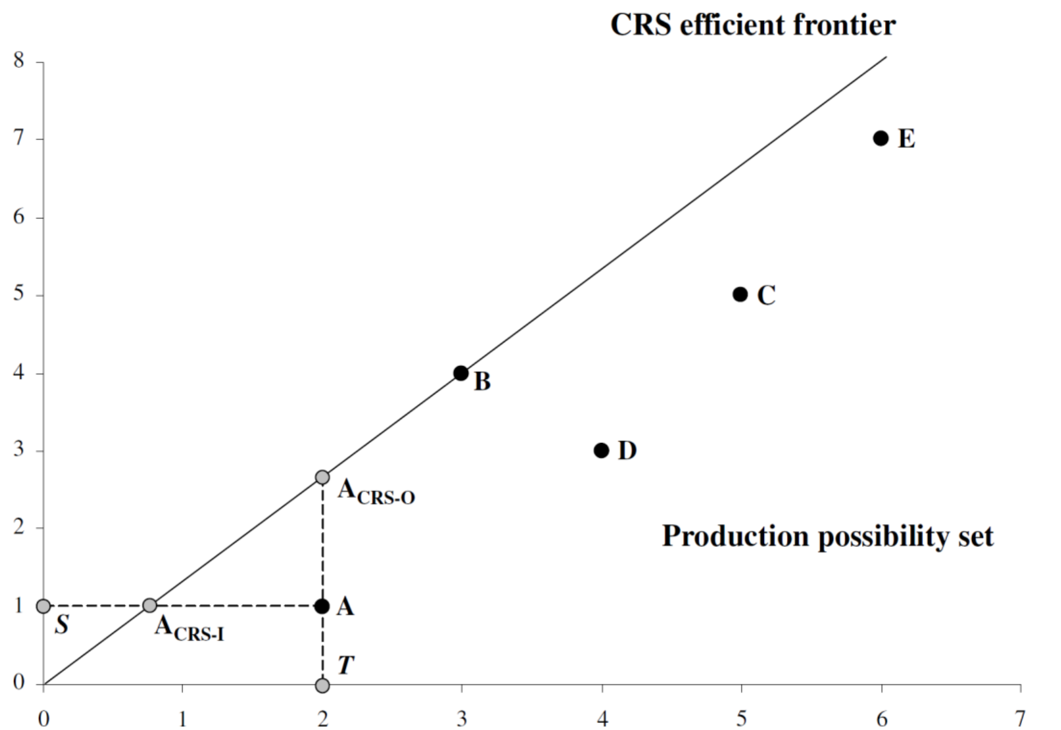
Data Envelopment Analysis

EXAMPLE

Technical Efficiency is measured by:

Distance(TA)/distance(TA $_{CRSI}$) = 1/(2.66) = 0.375

Technical Efficiency of A = 37.5%

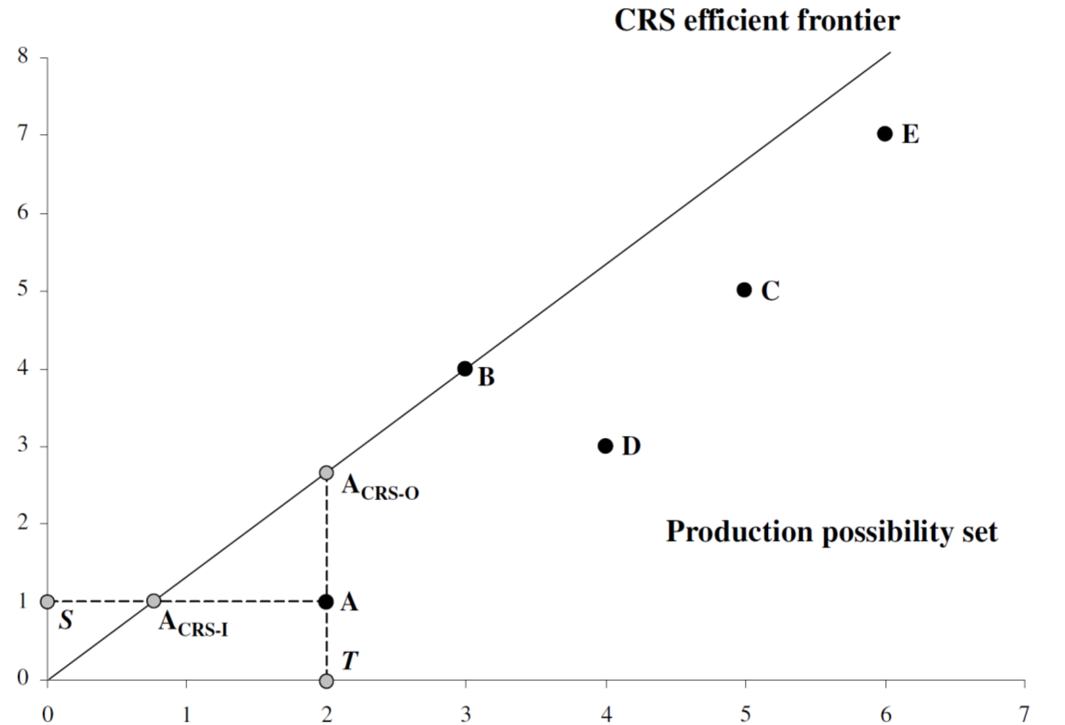


Data Envelopment Analysis

EXAMPLE

Technical output
Efficiency of A = 37.5%

It means that mine A could increase its production by 62.5% (100-37.5%) with the same number of workers



Data Envelopment Analysis

EXAMPLE of Variable Returns to Scale

In an Input orientation we are trying to minimize the input to produce the same level of output

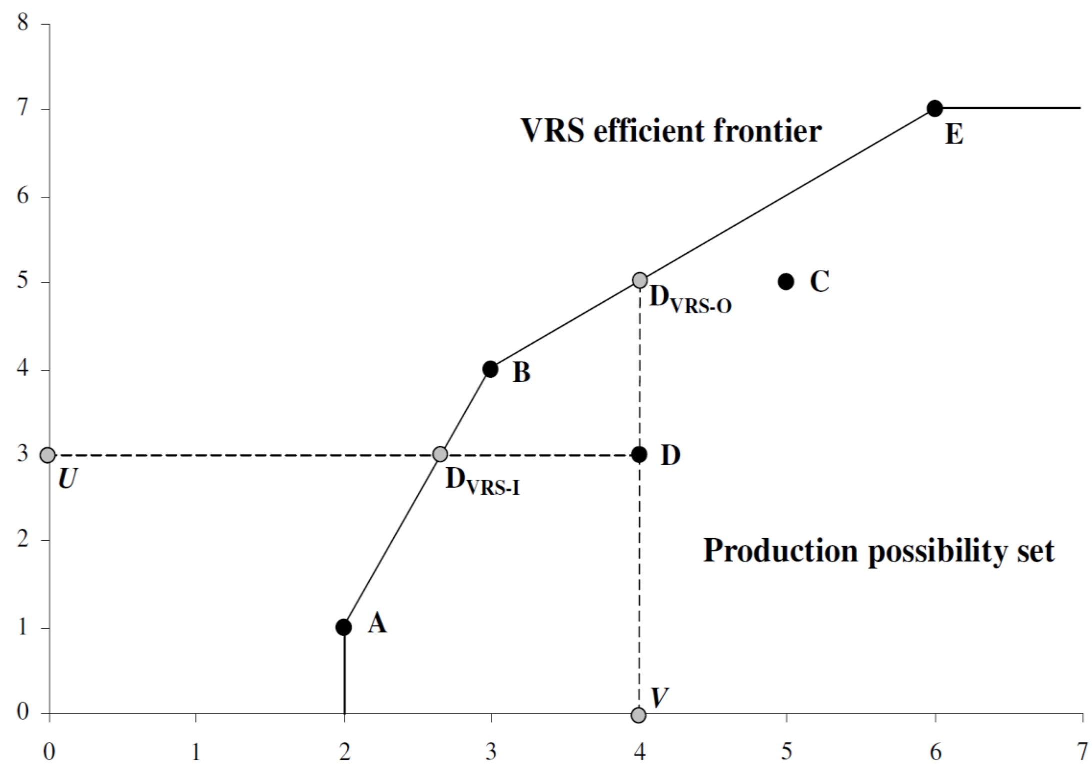
Data Envelopment Analysis

EXAMPLE

Technical Efficiency is measured by:

Distance(UD_{VRSI})/distance (UD) = 2.68/4 = 0.667

Technical Efficiency of D = 66.7%%

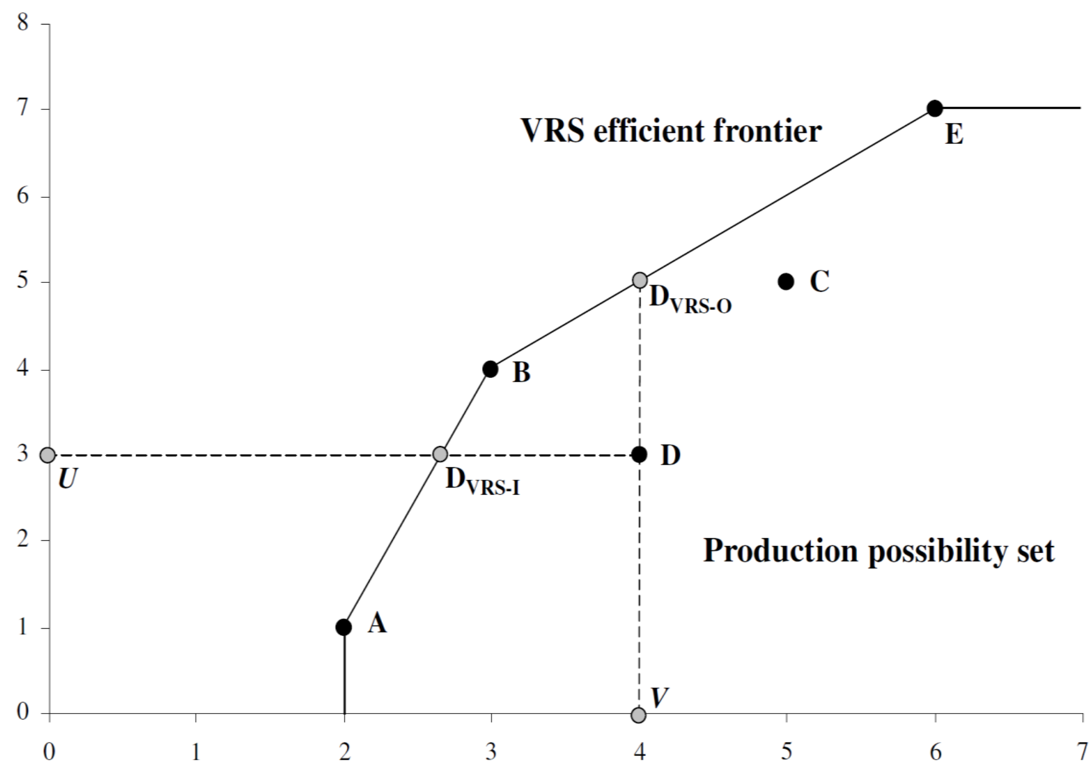


Data Envelopment Analysis

EXAMPLE

Technical Efficiency of D = 66.7%%

This means that mine D could reduce the number of workers by 33.3% (100-66.7%) and still produce the same output



Data Envelopment Analysis

EXAMPLE of Variable Returns to Scale

In an Output orientation we are trying to maximize the output by keeping the same level of input

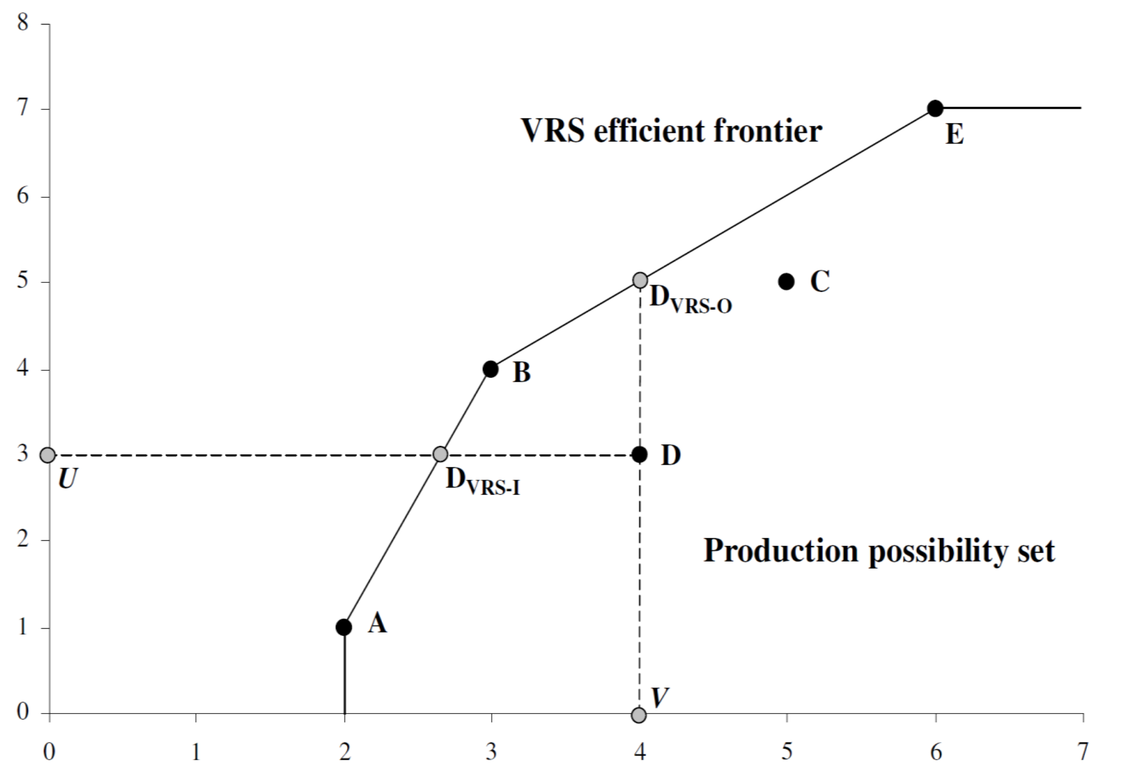
Data Envelopment Analysis

EXAMPLE

Technical output Efficiency is measured by:

Distance(VD)/distance(VA_{VRSO}) = 3/5 = 0.6

Technical Efficiency of A = 60%

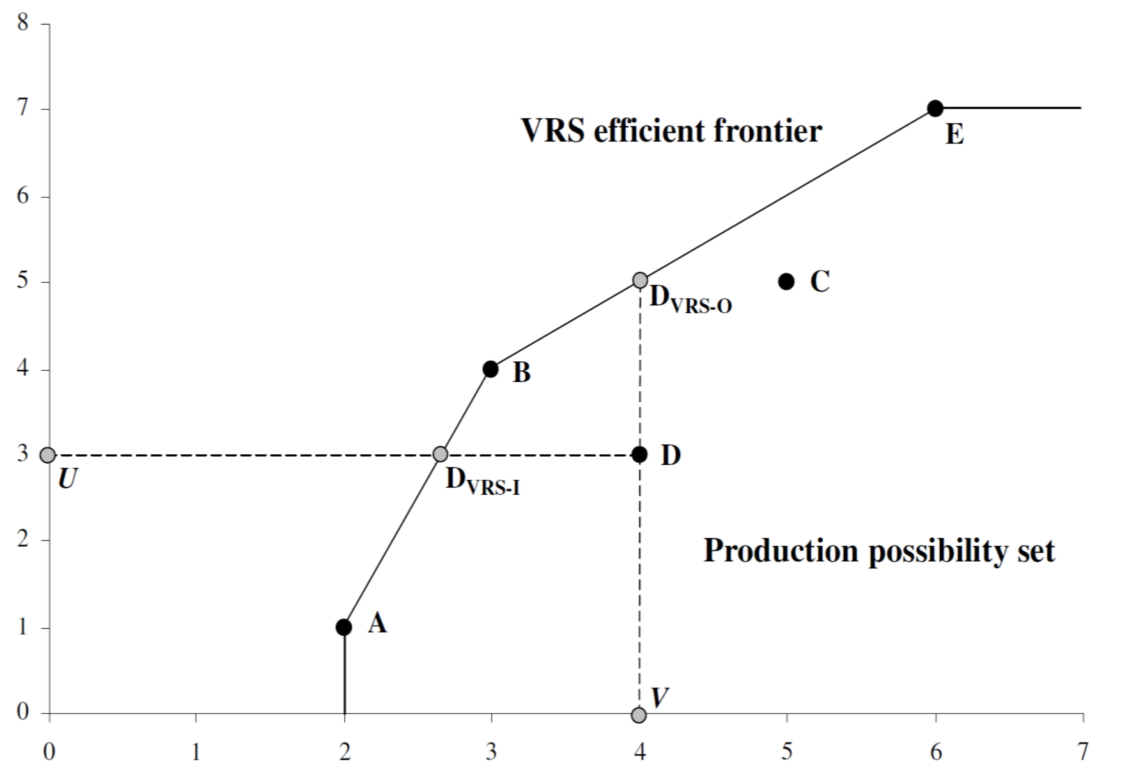


Data Envelopment Analysis

EXAMPLE

Technical Output output Efficiency of D = 60%

It means that mine D could increase its production by 40% (100-60%) with the same number of workers



Data Envelopment Analysis

EXAMPLE

- Mine A is located on the VRS frontier, but not on the CRS frontier. This indicates that the efficiency of mine A is because of **inappropriate scale**. A is facing increasing returns to scale, thus it must increase its output.
- Mine B is located both on the VRS and CRS frontiers, indicating that it is efficient.
- Mine C is located neither on the VRS nor on the CRS frontier, indicating that its inefficiency occurs due to **inappropriate scale and poor management**. Mine C is facing decreasing returns to scale.
- Mine D is located neither on the VRS nor on the CRS frontier, indicating that its inefficiency occurs due to **inappropriate scale and poor management**. Mine D is facing decreasing returns to scale.
- Mine E is located on the VRS frontier but not on the CRS frontier, indicating that the inefficiency of mine E occurs because of **inappropriate scale**. Mine E is facing decreasing returns to scale, thus it must decrease its output.

Data Envelopment Analysis

Questions

What is the main purpose of DEA?
1)It measures the technical efficiency of Decision Making Units?
2)It measures the productivity of Decision Making Units?
3)It measures the profitability of Decision Making Units?

How is efficiency measured?
1)By the ratio of input over output
2)By the ratio of output over input
3)By the ratio of weighted input over weighted output
4)By the ratio of weighted output over weighted input

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

Questions

In Constant Returns to Scale:

1) Output increases by the same proportional increase in input
2) Output decreases while input increases
3) Output increases while input decreases

Data Envelopment Analysis

Mathematical Formulation

Let's assume that there are N DMUs using n inputs to produce s outputs under Constant Returns to Scale (CRS). We denote x_{ij}(i=1…m, j=1…N) the level/value of the i^{th} input of DMU j, and y_{rj}(r= 1…s, j = 1…N) the level/value of DMU. Then the calculation for the *technical efficiency* can be found by solving the LP problem:

SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY
Co-funded by the Erasmus+ Programme of the European Union
ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY
SYstem Dynamics Italian Chapter
ERGO LVDO EDITIONS
CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA
60 YEARS 1957-2017
Societatea pentru Consum Responsabil
Centre for Systems Solutions

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

Mathematical Formulation

$$\text{minimize} \quad \Theta_0 - e\left(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+\right)$$

subject to the constraints:

$$\sum_{j=1}^{N} \lambda_j * x_{ij} = \Theta_0 * x_{ij_o} - S_i^-, i = 1 \dots m$$

$$\sum_{j=1}^{N} \lambda_j * y_{rj} = y_{rj_o} + S_r^+, r = 1 \dots s$$

$$\lambda_j \geq 0, j = 1 \dots N, S_r^+, S_i^- \geq 0$$

Data Envelopment Analysis

Mathematical Formulation

In Linear Optimization, a **slack variable** is one that is added to an inequality constraint to transform it into an equality.

In the solution of the problem:

- if a slack variable is zero then the constraint is binding
- if a slack variable is positive the constraint is non-binding
- if a slack variable is negative then the constraint is violated and the solution is not feasible

Co-funded by the Erasmus+ Programme of the European Union

GAME-BASED LEARNING ON URBAN SUSTAINABILITY

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

Mathematical Formulation

- DMUj_o is Pareto-efficient if and only if $\Theta_0{}^* = 1$ and $S_r^+, S_i^- = 0$ for all inputs and outputs.
- If one of the slack values is positive at the optimal solution of the model, it means that the corresponding input (or output) of DMUj_o can be further improved.
- If none of the above applies, then DMUj_o has technical input efficiency equal to $\Theta_0{}^*$.

If a solution is Pareto efficient, no changes can be made in one criterion without worsening any other

Data Envelopment Analysis

Other types of Efficiency

Technical Efficiency: Inputs and outputs are measured in physical terms

Cost Efficiency: Cost about inputs is added

Revenue Efficiency: Price information about outputs is added to the model

Profit Efficiency: Cost about inputs + Price information about outputs

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

Strengths and Weaknesses of DEA

1) DEA can handle multiple input and output models
2) It does not require an assumption of a functional form relating inputs to outputs
3) DMUs are directly compared against a combination of peers
4) It is unit invariant. Inputs and outputs can have different units

Data Envelopment Analysis

Strengths and **Weaknesses** of DEA

1) Noise (in data) can alter the results
2) It can not offer an "objective" evaluation of performance
3) Since it is a non-parametric technique, statistical hypothesis tests are difficult
4) Separate LP problem for each DMU can be computationally intensive in large problems

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

Implications for managers:

Assume that an efficiency score of 86.3% is obtained. Does this number have to be strictly applied??

Data Envelopment Analysis

Implications for managers:

Assume that an efficiency score of 86.3% is obtained. Does this number have to be strictly applied??

It should be interpreted more as an order of magnitude
Inform the managers that they have to increase output or decrease input to become more efficient

Data Envelopment Analysis

Implications for managers:

Should efficiency be the only criterion to assess a Unit's performance???

Data Envelopment Analysis

Implications for managers:

Should efficiency be the only criterion to assess a Unit’s performance???

Not necessarily. Other criteria could be considered, for example quality or effectiveness

Data Envelopment Analysis

Implications for managers:

Assume that all the scores of DMUs dataset are 1. What does that mean???

Data Envelopment Analysis

Areas of Application:

Health care (hospitals, doctors)
Education (Universities, schools)
Banks
Manufacturing
Benchmarking
Fast food restaurants
Energy efficiency of countries
Sustainability of countries

SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY
Co-funded by the Erasmus+ Programme of the European Union
ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ IKY

Data Envelopment Analysis

Exercise

A company operates 4 metro lines transporting people. The following table summarizes the data for one quarter.

Metro Line	Driver Hours (100 hr)	Operational Costs (10000 €)	Passengers (per 10000)
1	4.1	2.3	90
2	3.8	2.4	100
3	4.4	2.0	95
4	3.4	3.4	120

The inputs are driver hours and Operational costs while the output is the passengers serviced

SYSTEM DYNAMICS ITALIAN CHAPTER
ERGO LVDO EDITIONS
CERTH CENTRE FOR RESEARCH & TECHNOLOGY HELLAS
HELLENIC REPUBLIC UNIVERSITY OF MACEDONIA
60 YEARS 1957-2017
Societatea pentru Consum Responsabil
Centre for Systems Solutions

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

Exercise

$$\text{minimize} \qquad \Theta_1 - e\left(S^-_{drivers} + S^-_{Cost} + S^+_{Passengers}\right)$$

$$\textit{subject to the constraints}:$$

$$4.1 * \Theta_1 - S^-_{drivers} = 4.1 * \lambda_1 + 3.8 * \lambda_2 + 4.4 * \lambda_3 + 3.4 * \lambda_4$$
$$2.3 * \Theta_1 - S^-_{Costs} = 2.3 * \lambda_1 + 2.4 * \lambda_2 + 2.0 * \lambda_3 + 3.4 * \lambda_4$$
$$90 + S^+_{Passengers} = 90 * \lambda_1 + 100 * \lambda_2 + 95 * \lambda_3 + 120 * \lambda_4$$
$$\lambda_j \geq 0, j = 1,2,3,4$$

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

How to solve it in EXCEL

G11

	A	B	C	D	E
1	Metro Lines	Driver Hours	Operational Costs	Passengers	
2	1	4.1	2.3	90	
3	2	3.8	2.4	100	
4	3	4.4	2	95	
5	4	3.4	3.4	120	
6					
7					
8					
9					

Data Envelopment Analysis

How to solve it in EXCEL

A13 | Constraints

	A	B	C	D	E	F	G	H
1	Metro Lines	Driver Hours	Operational Costs	Passengers	λ			
2	1	4.1	2.3	9	1			
3	2	3.8	2.4	10	0			
4	3	4.4	2	9.5	0			
5	4	3	3.4	12	0			
6								
7						DMU under evaluation	1	Efficiency
8								0

Data Envelopment Analysis

How to solve it in EXCEL

	A	B	C	D	E	F	G	H
1	**Metro Lines**	**Driver Hours**	**Operational Costs**	**Passengers**	**λ**			
2	1	4.1	2.3	9	1			
3	2	3.8	2.4	10	0			
4	3	4.4	2	9.5	0			
5	4	3	3.4	12	0			
6								
7						DMU under evaluation	1	Efficiency
8	**Constraints**							0
9	Driver Hours	4.1	<=	0				
10	Operational Costs	2.3	<=	0				
11	Passengers	9	>=	9				
12	Σλ	1	=	1				
13								
14								
15								

Data Envelopment Analysis

How to solve it in EXCEL

In the respective cells we set:
Cell B9: =SUMPRODUCT(B2:B5,$E$2:$E$5)
Cell B10: =SUMPRODUCT(C2:C5,$E$2:$E$5)
Cell B11: =SUMPRODUCT(D2:D5,$E$2:$E$5)
Cell B12: =SUM(E2:E5)

Data Envelopment Analysis

How to solve it in EXCEL

Then, we define the right-hand sides of the constraints as following:
Cell D9: =$H$8*INDEX(B2:B5,G7,1)
Cell D10:=$H$8*INDEX(C2:C5,G7,1)
Cell D11:=INDEX(D2:D5,G7,1)
Cell D12: =1

Data Envelopment Analysis

How to solve it in EXCEL

Finally, to solve the model we need to use Excel's solver add-in. To Do so:

Click the **File** tab and then click **Options**

Click **Add-ins** and the in the **Manage** box select the **Excel Add-ins** option

Click **Go**

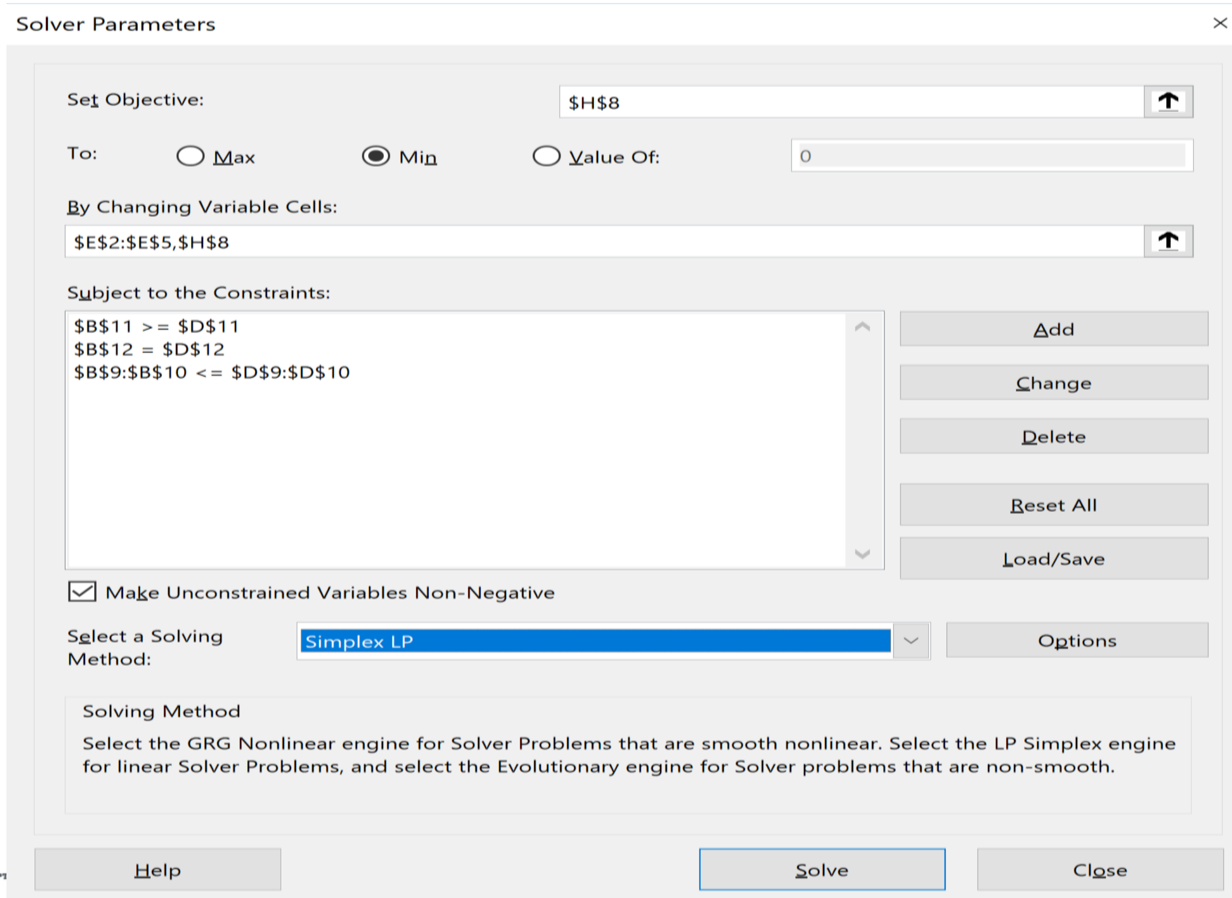
In the Add-ins available,select the Solver Add-in and then OK.

Go to **Data** tab and select the Solver option.

Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

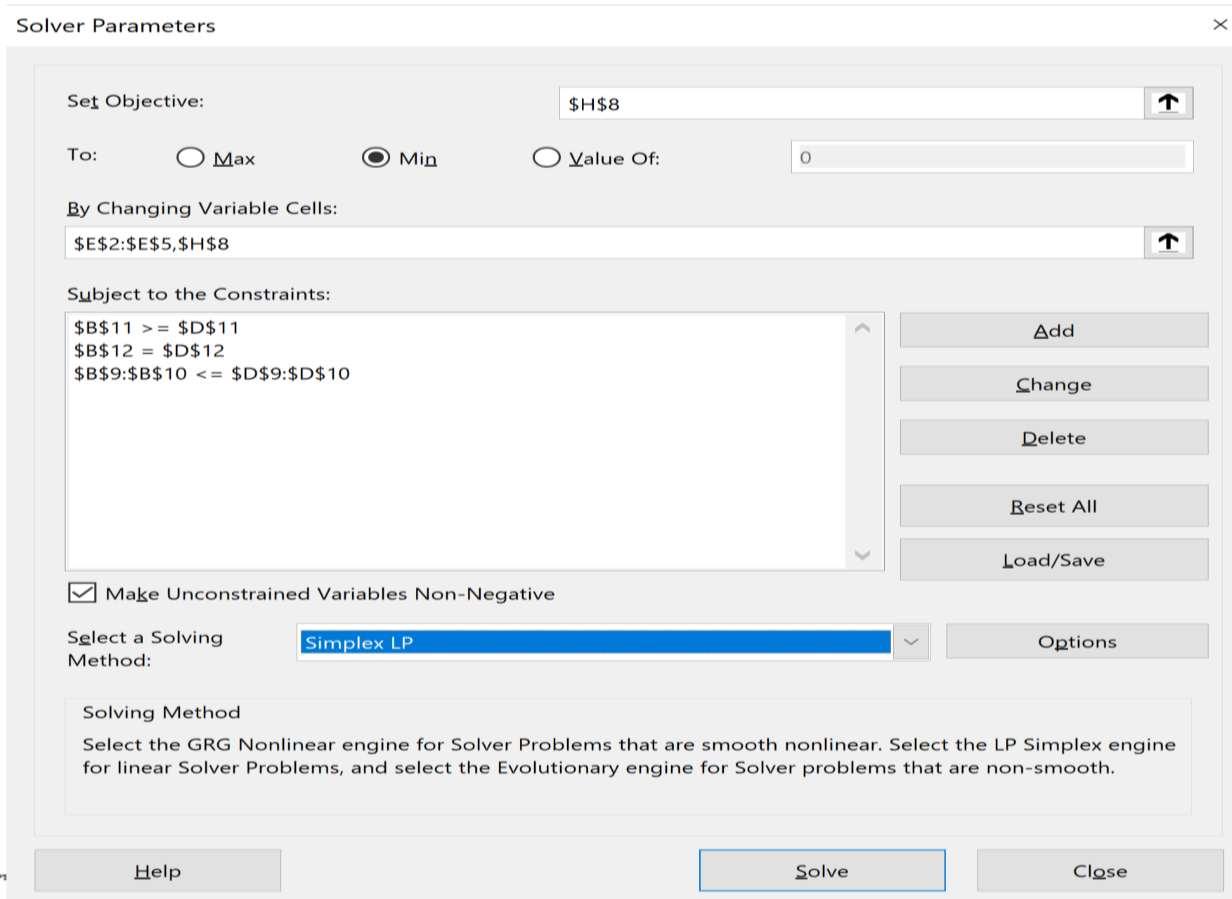
How to solve it in EXCEL



Co-funded by the Erasmus+ Programme of the European Union

Data Envelopment Analysis

How to solve it in EXCEL



Data Envelopment Analysis

How to solve it in EXCEL

After you have copied the values above, press Solve. The solution to the model is displayed in cell H8.

Data Envelopment Analysis

Implications for managers:

Assume that all the scores of DMUs dataset are 1. What does that mean???

1) All the DMUs are equally efficient!!!

Data Envelopment Analysis

Implications for managers:

Assume that all the scores of DMUs dataset are 1. What does that mean???

1) All the DMUs are equally efficient!!!
2) The number of inputs and outputs is too high for the number of DMUs that are compared

Co-funded by the Erasmus+ Programme of the European Union

SUSTAIN GAME-BASED LEARNING ON URBAN SUSTAINABILITY

Co-funded by the Erasmus+ Programme of the European Union

ANY QUESTIONS?